File S1

S.1. Geometric Model

To accurately represent the Earth's geometry, we adopt a rotational ellipsoid instead of a simple sphere. The CGCS2000 reference ellipsoid provides the geometric foundation for all subsequent derivations.

S.1.1 Core Parameters of the CGCS2000 Ellipsoid

The CGCS2000 ellipsoid is characterized by the following parameters:
Semi-major axis: $a = 6378137.0m$
Flattening: $f = 1/298.257222101$
First eccentricity squared: $e^2 = 2f - f^2 \approx 0.00669438002290$
These parameters are essential for determining geometric properties of the ellipsoid.

S.1.2 Geodetic Coordinate System

We adopt geodetic coordinates to describe locations on the ellipsoid:
Geodetic latitude: $B \in [-\pi/2, \pi/2]$
Longitude: $L \in [0,2\pi]$

S.1.3 Surface Area Element on the Ellipsoid

When integrating over an ellipsoid, the commonly used spherical element $R2sin\theta$ is no longer valid. The differential area dS depends on the **principal radii of curvature**, which vary with latitude.

Meridian radius of curvature: $M = a(1-e^2)/\left(1-e^2sin^2(B)\right)^{(3/2)}$
Prime-vertical radius of curvature: $N = a/sqrt\left(1-e^2sin^2(B)\right)$
The surface area element on the ellipsoid is therefore:

$$dS = M * N * cos(B) * dB * dL \tag{S1}$$

Expanding in terms of only $a, e, and\ B$, we obtain:

$$dS = a^2(1-e^2)cos(B)/\left(1-e^2sin^2(B)\right)^2 * dB * dL \tag{S2}$$

For convenience, we define the latitude-dependent weight term:

$$W(B) = a^2(1-e^2)cos(B)/\left(1-e^2sin^2(B)\right)^2, dS = W(B) * dB * dL \tag{S3}$$

Total surface area of the ellipsoid:

$$S_{total} = \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} W(B)dB\,dL \tag{S4}$$

S.2. Physical Process

This section describes how the temperature field evolves over time. Unlike simple homogenization, we refine the physical interpretation: random temperature anomalies (δu) diffuse and decay over time, while the deterministic climatic background (μ_{clim}) remains unchanged.

S.2.1 Definitions of Physical Quantities

To precisely describe this process, we decompose the temperature field into a deterministic background and a random perturbation:
Temperature field decomposition (Word formula style):

$$u(\boldsymbol{r}, t) = \mu_{clim}(\boldsymbol{r}) + \delta u(\boldsymbol{r}, t) \tag{S5}$$

$\mu_{clim}(\boldsymbol{r})$: long-term climatic background, maintained by solar radiation and Earth's geometry (e.g., warm equator, cold poles). Does not decay over time.

$\delta u(\boldsymbol{r}, t)$: temperature anomaly (random perturbation), caused by weather systems or measurement deviations.

Initial observed field at time $t = t_1$:

$$u_0(\boldsymbol{r}) = \mu_{clim}(\boldsymbol{r}) + \delta u_0(\boldsymbol{r}) \tag{S6}$$

Among them, $\delta u_0(\boldsymbol{r})$ is the random anomaly field at the initial moment, anomaly field expected value: $E[\delta u_0(\boldsymbol{r})] = 0$.

Initial covariance function, describe the spatial correlation of the anomaly field (i.e. the source of randomness we are concerned about):

$$C_\delta(r, r') = Cov[\delta u_0(\boldsymbol{r}), \delta u_0(\boldsymbol{r}')] \tag{S7}$$

S.2.2 Dynamic Equation

Diffusion acts only on δu; μ_{clim} remains fixed:

The thermal diffusion equation only applies to the random anomaly δu. The background climate μ_{clim} is considered a steady state maintained by the energy source term and does not participate in attenuation.

$$\partial(\delta u)/\partial t = (\alpha/R^2)\nabla^2_{sph}(\delta u) \tag{S8}$$

α: thermal diffusivity;

The equation describes local warm or cold anomalies spread and decay over time. The boundary condition is periodic closed surface of the CGCS2000 ellipsoid surface. Ideal diffusion will maintain energy dynamic balance (conservation of anomaly), which means that the diffusion process does not change the total heat deviation of the entire region:

$$\overline{\delta u}(t) = (1/S_{total}) \int_\Omega \delta u(\boldsymbol{r}, t) dS \equiv \overline{\delta u_0} \tag{S9}$$

S.2.3 Spectral Decomposition of the Dynamic Relaxation Process

Expand δu in spherical harmonics to show return to the background climate.

S.2.3.1 Series Representation

$$\delta u(\theta, \varphi, t) = \Sigma_{l=0}^{\infty} \Sigma_{m=-l}^{l} b_{lm}(t_1) \cdot exp[-\alpha \lambda_l (t - t_1)/R^2] \cdot Y_l^m(\theta, \varphi) \tag{S10}$$

b_{lm}: expansion coefficients of the initial anomaly δu_0

S.2.3.2 Asymptotic Behavior

Case 1: $l \geq 1$ (local modes)

$$lim_{t \to \infty} exp[-\alpha \lambda_l t/R^2] = 0 \tag{S11}$$

Meaning: all local cold and warm anomalies (such as cold waves or heat waves in a certain area) will be smoothed out as they spread.

Case 2: $l = 0$ (global mean mode)

$$lim_{t \to \infty} exp[0] = 1 \tag{S12}$$

Only the global average anomaly deviation $\overline{\delta u_0}$ will be retained.

S.2.3.3 Fixed-Point Temperature Field

As t increases, the local anomaly disappears, leaving only the global average anomaly. Residual anomaly:

$$\delta u_\infty = lim_{t \to \infty} \delta u(\boldsymbol{r}, t) = \overline{\delta u_0} = (1/S_{total}) \int_\Omega \delta u_0(\boldsymbol{r}) dS \tag{S13}$$

Final temperature field $u_\infty(\boldsymbol{r})$:

$$u_\infty(\boldsymbol{r}) = \mu_{clim}(\boldsymbol{r}) + \overline{\delta u_0} \tag{S14}$$

The fixed-point temperature field is not spatially uniform. It restores the climatic background$\mu_{clim}(\boldsymbol{r})$ with only a globally uniform residual anomaly $\overline{\delta u_0}$.

S.3. Diffusion Model Based on Temperature Anomalies

The physical model assumes that the initial observed temperature field $u_0(\boldsymbol{r})$ is decomposed as: $u_0(\boldsymbol{r}) = \mu_{clim}(\boldsymbol{r}) + \delta u_0(\boldsymbol{r})$.

$\mu_{clim}(\boldsymbol{r})$: deterministic background (long-term climate state, e.g., high at the equator, low at the poles). Maintained by external forcing (solar radiation) and does not decay in the diffusion equation (In other words, it is itself the steady-state solution of the active diffusion equation).

$\delta u_0(\boldsymbol{r})$: instantaneous random fluctuation during observation (e.g., gusts, cloud-induced temperature differences). This is a random field:

$$E[\delta u_0(\boldsymbol{r})] = 0 \tag{S15}$$

New dynamic mechanism: Only the random perturbation $\delta u(\boldsymbol{r}, t)$ obeys the source-free ideal diffusion equation:

$$\partial(\delta u)/\partial t = \alpha \nabla^2 \delta u \tag{S16}$$

This implies that as t increases, local random fluctuations diffuse and converge to a spatially uniform global mean.

S.3.1 Mean Derivation

Step 1: Construct the fixed-point temperature field

As t increases, δu diffuses into a spatially uniform global average anomaly perturbation $\overline{\delta u_0}$:

$$lim_{t\to\infty}\delta u(\boldsymbol{r}, t) = (1/S_{total}) \iint_\Omega \delta u_0(r')dS' \equiv \overline{\delta u_0} \tag{S17}$$

Hence, the final fixed-point temperature field is:

$$u_\infty(\boldsymbol{r}) = \mu_{clim}(\boldsymbol{r}) + \overline{\delta u_0} \tag{S18}$$

Step 2: Compute the expectation $E[u_\infty(r)]$

$$E[u_\infty(\boldsymbol{r})] = E\left[\mu_{clim}(\boldsymbol{r}) + \overline{\delta u_0}\right] \tag{S19}$$

Using linearity of expectation:

$$E[u_\infty(\boldsymbol{r})] = \mu_{clim}(\boldsymbol{r}) + E\left[\overline{\delta u_0}\right] \tag{S20}$$

Step 3: Analyze the expectation of residual perturbation

$$E\left[\overline{\delta u_0}\right] = E\left[(1/S_{total}) \iint_\Omega \delta u_0(\boldsymbol{r}')dS'\right] \tag{S21}$$

Exchange expectation and integral:

$$E\left[\overline{\delta u_0}\right] = (1/S_{total}) \iint_\Omega E[\delta u_0(\boldsymbol{r}')]dS' \tag{S22}$$

Since the initial perturbation is zero-mean:

$$E\left[\overline{\delta u_0}\right] = (1/S_{total}) \iint_\Omega 0\, dS' = 0 \tag{S23}$$

Corrected Result S.3.1: The expected fixed-point temperature equals the long-term background at each point:

$$E[u_\infty(B,L)] = \mu_{clim}(B,L) \tag{S24}$$

After long-term diffusion, the temperature at any point may fluctuate, but its probability distribution center (expectation) remains anchored to the local climate background.

S.3.2 Variance Derivation

Although the expectation returns to the local climate, the fixed-point temperature still has fluctuations because $\overline{\delta u_0}$ is a random variable.

Step 1: Define variance

$$Var[u_\infty(\boldsymbol{r})] = Var\left[\mu_{clim}(\boldsymbol{r}) + \overline{\delta u_0}\right] \tag{S25}$$

Since $\mu_{clim}(r)$ is deterministic, it does not contribute variance:

$$Var[u_\infty(\boldsymbol{r})] = Var\left[\overline{\delta u_0}\right] \tag{S26}$$

Step 2: Variance of spatial average

$$Var\left[\overline{\delta u_0}\right] = Var\left[(1/S_{total})\iint_\Omega \delta u_0(\boldsymbol{r}')dS'\right] \tag{S27}$$

Using the properties of double integrals and the covariance of δu_0:

$$Var\left[\overline{\delta u_0}\right] = (1/S_{total}^2)\iint_\Omega \iint_\Omega Cov[\delta u_0(\boldsymbol{r}'), \delta u_0(\boldsymbol{r}'')]dS'dS'' \tag{S28}$$

dS: the generic surface differential element on the ellipsoid. dS': the surface differential element at position $\boldsymbol{r}'$ (Location A). dS'': the surface differential element at position $\boldsymbol{r}''$ (Location B).

Step 3: Substitute covariance function in CGCS2000 coordinates

$$\sigma_\infty^2 = (1/S_{total}^2)\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} C_\delta(B,L;B',L')\cdot W(B)W(B')dBdLdB'dL' \tag{S29}$$

The fixed-point variance σ_∞^2 is spatially constant. This means the expected value varies with latitude, but the amplitude of residual random error is uniform globally. It depends on the global total correlation of the initial random disturbance.

S.4. Central Limit Theorem Effect (Rigorous Mathematical Proof from Non-Gaussian to Gaussian)

Objective: Prove that the globally averaged residual $\overline{\delta u_0}$ of a locally non-Gaussian temperature anomaly field $\delta u_0(\boldsymbol{r})$ defined on the CGCS2000 ellipsoid strictly converges to a Gaussian distribution.

$$\overline{\delta u_0} = (1/S_{total})\iint_\Omega \delta u_0(\boldsymbol{r})dS \tag{S30}$$

S.4.1 Definitions and Preliminaries

Core random variable X:

$$X = \overline{\delta u_0} = (1/S_{total})\iint_\Omega \delta u_0(\boldsymbol{r})dS \tag{S31}$$

Ω: CGCS2000 ellipsoidal surface; S_{total}: total ellipsoid surface area; dS: area element of the ellipsoid, $dS = W(B)dBdL$.

Non-Gaussianity of the initial field: Assume $\delta u_0(\boldsymbol{r})$ is non-Gaussian locally, i.e., higher-order cumulants do not vanish.

For example, the third-order cumulant (skewness) may exist:

$$Cum[\delta u_0(r_1), \delta u_0(r_2), \delta u_0(r_3)] \neq 0 \tag{S32}$$

Mixing condition (weak correlation):

Assume a correlation length $\ell_c \ll \sqrt{S_{total}}$, when the distance between the ground lines of two points on the ellipsoid is $d(\boldsymbol{r}_i, \boldsymbol{r}_j) \gg \ell_c$, the correlation function rapidly decays.

$$\left|Cov[\delta u_0(\boldsymbol{r}_i), \delta u_0(\boldsymbol{r}_j)]\right| \leq Aexp(-d(\boldsymbol{r}_i, \boldsymbol{r}_j)/\ell_c) \tag{S33}$$

This implies the Earth's surface contains a large number of approximately (about $N_{eff} \approx S_{total}/(\pi\ell_c^2)$) independent "microclimate regions".

S.4.2 Cumulant Generating Function (CGF) and Goal

Consider the cumulant generating function of X:

$$K_X(t) = lnE[e^{tX}] = \sum_{k=1}^{\infty}(t^k/k!)\kappa_k(X) \tag{S34}$$

$\kappa_k(X)$ is the k-th cumulant of X; For a Gaussian, all κ_k with $k \geq 3$ must vanish.

Proof idea: the higher-order cumulants of the standardized variable approach zero. Then, by the inversion of the characteristic function, the distribution converges to Gaussian.

S.4.3 k-th Order Cumulant of X

Due to the multilinearity of cumulants:

$$\kappa_k(X) = \left(1/S_{total}^k\right)\iint_{\Omega}\dots\iint_{\Omega} Cum[\delta u_0(\boldsymbol{r}_1), \dots, \delta u_0(\boldsymbol{r}_k)]dS_1 \dots dS_k \tag{S35}$$

Define the k-point correlation density function of δu_0 as $C_k(r_1, \dots, r_k)$.

$$\kappa_k(X) = \left(1/S_{total}^k\right)\int_{\Omega^k} C_k(\boldsymbol{r}_1, \dots, \boldsymbol{r}_k)dS^k \tag{S36}$$

S.4.4 Scaling Analysis

Due to the rapid convergence of ℓ_c to zero when the variable space separation exceeds ℓ_c, the effective contribution of the k-th integration mentioned above only comes from all points $(\boldsymbol{r}_1, \dots, \boldsymbol{r}_k)$ the situation where $gathers in a small area $\sim\ell_c^2$.

Approximate integral:

$$\int_{\Omega^k} C_k dS^k \approx \iint_{\Omega} dS_1\left(\int \dots \int C_k dS_2 \dots dS_k\right) \approx S_{total} \cdot \ell_c^{2(k-1)} \tag{S37}$$

Then key attenuation law:

$$\kappa_k(X) \approx \left(1/S_{total}^k\right)\left[S_{total} \cdot \ell_c^{2(k-1)}\right] \propto S_{total}^{-(k-1)} \tag{S38}$$

Variance (k=2): $\kappa_2 \propto S_{total}^{-1}$; Skewness (k=3): $\kappa_3 \propto S_{total}^{-2}$.

S.4.5 Standardized Variable Limit

Define standardized variable Z:

$$Z = (X - E[X])/\sqrt{Var[X]} = X/\sqrt{\kappa_2(X)} \tag{S39}$$

Its k-th cumulant of Z, (enter formula 38):

$$\tilde{\kappa}_k(Z) = \kappa_k(X)/[\kappa_2(X)]^{k/2} \propto S_{total}^{-(k-1)}/[S_{total}^{-1}]^{k/2} = S_{total}^{1-k/2} \tag{S40}$$

k=2 (variance): $\tilde{\kappa}_2 \sim 1$; k $\geq$ 3 (higher-order cumulants): $1 - k/2 < 0$, thus $\lim_{S_{total}\to\infty} \tilde{\kappa}_k(Z) = 0$.

S.4.6 Characteristic Function Convergence

The logarithmic expansion of the characteristic function $\varphi_Z(t)$ of the random variable Z:

$$ln\varphi_Z(t) = \sum_{k=1}^{\infty} (it)^k / k! \cdot \tilde{\kappa}_k \tag{S41}$$

In the limit $S_{total} \to \infty$:

$$ln\varphi_Z(t) \to (it)^2/2! \cdot 1 + 0 + \cdots = -t^2/2 \tag{S42}$$

The characteristic function converges:

$$\varphi_Z(t) = e^{-t^2/2} \tag{S43}$$

By Lévy's continuity theorem, the cumulative distribution function of the random variable Z must converge to the standard normal distribution $\sim N(0,1)$.

In a word, regardless of the local non-Gaussianity of δu_0, the global average anomaly residual follows a Gaussian distribution.

S.5. Final Conclusion: Fixed-Point Distribution

Based on the CGCS2000 ellipsoid and the temperature anomaly diffusion mechanism, as t increases, the temperature at any fixed point (B, L) satisfies the Gaussian distribution (proved by the central limit theorem, applied to $\overline{\delta u_0}$).

S.5.1 Distribution Type

$$u_\infty(B, L) \sim N(\mu_{clim}(B, L), \sigma_\infty^2) \tag{S44}$$

S.5.2 Probability Density Function (PDF)

$$f(u; B, L) = \left(1/\sqrt{(2\pi\sigma_\infty^2)}\right) exp\left(-\left(u - \mu_{clim}(B, L)\right)^2/(2\sigma_\infty^2)\right) \tag{S45}$$

This indicates that the final temperature value fluctuates around the climate background value μ_{clim} of the location, with the magnitude of the fluctuation determined by the global residual noise σ_∞, and the probability shape of the fluctuation follows a standard normal curve.

S.5.3 Statistical Parameters

Expectation (anchored at local climate):

$$E[u_\infty(B, L)] = \mu_{clim}(B, L) \tag{S46}$$

This represents a deterministic background field (such as the temperature difference between the equator and the poles) that will not be smoothed out by diffusion.

Variance (global residual noise, spatially constant):

$$\sigma_\infty^2 = (1/S_{total}^2) \iint_\Omega \iint_\Omega C_\delta(P, P') dS_P dS_{P'} \tag{S47}$$

The variance is determined by the spatial correlation of the initial δu_0. If there are no additional heat sources or sinks in the ideal diffusion model, their values are the same at any point in the world.

S.6. Diagnostic Framework for Non-Ideal Diffusion with Transient Heat Sources

We demonstrated that under ideal diffusion without external forcing, both the temperature field u and its spatial averages follow Gaussian statistics. Building on this statistical baseline, we develop an anomaly-detection framework applicable to non-ideal scenarios involving transient heat sources Q. The key concept is straightforward: any statistically significant deviation from the ideal Gaussian structure can be attributed to the intervention of a physical sudden heat source.

S.6.1 Physical Equivalence

When transient subsurface heating or energy imbalance exists, the heat-conduction equation becomes:

$$\rho c_p \frac{\partial u}{\partial t} = k\nabla^2 u + Q(\mathbf{r}, t) \tag{S48}$$

where Q is an unknown volumetric heat source, and ρ, c_p, and k denote density, specific heat capacity, and thermal conductivity, respectively.

To diagnose the presence of Q, we define a diagnostic residual term $\Delta(\mathbf{r}, t)$:

$$\Delta(\mathbf{r}, t) = \frac{\partial u}{\partial t} - \alpha\nabla^2 u \tag{S49}$$

$\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity.

Substituting the physical equation yields the identity:

$$\Delta(\boldsymbol{r}, t) = \frac{Q(\boldsymbol{r}, t)}{\rho c_p} \tag{S50}$$

Thus, monitoring Δ is mathematically equivalent to estimating the normalized heat-source strength. If $\Delta \approx 0$: the system follows ideal diffusion equilibrium. If Δ shows significant departures from zero: an external heat source must be present.

S.6.2 Statistical Basis

In practical observations, the measured temperature field contains independent Gaussian noise:

$$u_{obs} = u_{true} + \varepsilon, \varepsilon \sim \mathcal{N}(0, \sigma_u^2) \tag{S51}$$

Because temporal and spatial differential operators are linear, the residual Δ is also a linear transformation of a Gaussian variable. According to probability theory, a linear combination of Gaussian distributions is still Gaussian distribution.

We establish the null hypothesis:

H_0: $Q = 0$, there is no external heat source.

Under the null condition, the diagnostic residual follows a zero-mean Gaussian distribution:

$$\Delta_{null} \sim \mathcal{N}(0, \sigma_\Delta^2) \tag{S52}$$

This provides the theoretical foundation for the 3σ detection rule: values outside $[-3\sigma_\Delta, +3\sigma_\Delta]$ can be rejected with 99.7% confidence.

S.6.3 Descriptive Metrics

To determine the detection threshold (i.e. the theoretical variance of residuals), we adopt a data-driven approach. This method comprehensively evaluates the volatility and interrelationships of time and space terms by analyzing observation data within a certain spatiotemporal domain, to estimate the theoretical value in formula 47.

According to the definition of residuals, the uncertainty of residuals is determined by the joint distribution of the "time variation term" and the "spatial diffusion term".

To describe the spatiotemporal characteristics of diagnostic residual term $\Delta(\mathbf{r}, t)$, we define the following core indicators to describe uncertainty in the spatiotemporal domain.

Temporal Uncertainty:

$$\sigma_t = std\left(\frac{\partial u}{\partial t}\right) \tag{S53}$$

Reflects the background fluctuation of the temporal derivative.

Spatial Uncertainty:

$$\sigma_s = std(\alpha\nabla^2 u) \tag{S54}$$

Captures the background variability of spatial diffusion.

Correlation Coefficient:

$$\rho = Corr\left(\frac{\partial u}{\partial t}, \alpha\nabla^2 u\right) \tag{S55}$$

Describes the linear dependence between temporal and spatial contributions.

Spatiotemporal Uncertainty (Final Threshold):

Using the variance formula $Var(A-B) = Var(A) + Var(B) - 2Cov(A,B)$, we obtain:

$$\sigma_{st}^2 = \sigma_t^2 + \sigma_s^2 - 2\rho\sigma_t\sigma_s \tag{S56}$$

This variance represents the theoretical fluctuation range of Δ in non-anomalous background conditions.

In a word, Temporal uncertainty can be derived from long-term background time series at a fixed location, ensuring robustness of temporal fluctuations. Spatial uncertainty can be estimated from temperature variability across a spatial neighborhood at a fixed time, capturing the spatial stability of the field. The spatiotemporal correlation coefficient is then introduced to correct the overall uncertainty and reflect the coupling between temporal and spatial deviations.

The spatiotemporal correlation coefficient fundamentally quantifies the coupling between temporal and spatial variations within the target region and thus should be estimated based on statistical analysis over a sufficiently long-time window. Interestingly, a similar concept appears in existing thermal anomaly detection algorithms as the heat penetration index defined by $D = 1 - |\rho_{XY}|$. This index characterizes the response of different heat-conducting media to a heat source, distinguishing whether the source originates from the atmosphere or the subsurface. Specifically, a larger subsurface thermal anomaly corresponds to a higher heat penetration index, a lower correlation coefficient, and greater independence between temporal and spatial uncertainties. Consequently, the combined spatiotemporal uncertainty more accurately reflects the true variability of surface temperature.

S.6.4 Final Diagnostic Criterion

The complete anomaly-detection algorithm consists of:

1. Parameter estimation: Use background or historical data to compute σ_t, σ_s, and ρ, then derive the threshold σ_{st}.
2. Residual computation: Evaluate $\Delta(\boldsymbol{r}, t)$ from real-time observations.
3. Anomaly classification using the 3σ rule:

$$|\Delta(\boldsymbol{r}, t)| > 3\sigma_{st} \tag{S57}$$

If the inequality holds, it is judged as an abnormal heat source event; otherwise, it is judged as background fluctuation (normal).

S. 7 Theoretical Unification of Existing Algorithms

The proposed framework offers a generalized perspective, mathematically encompassing existing algorithms as degenerate cases under specific boundary conditions.

S.7.1 Case I: Temporal Dominance (Equivalence to Z-Score / RST)

Assuming the region is spatially homogeneous ($\sigma_Y \to 0$) or the analysis ignores spatial diffusion, and assuming spatiotemporal decoupling ($\rho_{XY} \to 0$), Eq. (S57) simplifies to:

$$\sigma_\Delta \approx \sigma_X = \mathrm{std}\big((\partial u)/(\partial t)\big) \tag{S58}$$

In this regime, the detection criterion degenerates to $|u - \mu| > k\sigma_X$, which is mathematically equivalent to the Z-Score method. When an iterative outlier removal process is applied to estimate robust μ and σ_X, this formulation aligns with the Robust Satellite Techniques (RST). These methods implicitly assume that thermal anomalies manifest exclusively as temporal deviations, neglecting spatial heat transport.

S.7.2 Case II: Spatial Dominance (Equivalence to Eddy Field / Gradient)

Conversely, assuming a steady-state temporal field ($\sigma_X \to 0$) or neglecting temporal evolution, the joint uncertainty becomes:

$$\sigma_\Delta \approx \sigma_Y = \alpha \cdot \mathrm{std}(\nabla^2 u) \tag{S59}$$

This leads to detection based on the Laplacian or gradient modulus of the temperature field, which is consistent with Eddy Field or Thermal Texture analysis methods. These approaches focus solely on morphological anomalies, ignoring temporal dynamics.

S7.3 Case III: Spatiotemporal Coupling(Heat penetrability index)

In realistic seismic scenarios, thermal anomalies involve both temporal warming and spatial diffusion. The proposed model retains the interaction term:$-2\rho_{XY}\sigma_X\sigma_Y$.

When ρ_{XY} is high, the term reduces σ_Δ, tightening the threshold to suppress background noise.

When ρ_{XY} is low, the model adaptively adjusts the sensitivity to capture transient signals that traditional decoupled methods cannot detect.

At the same time, this rho is consistent with the definition of the heat penetrability index[1], which is a thermal anomaly description index in the literature. Used to explain that the difference in heat may come from underground or above ground sources.

Reference

1. Liu, P.; Liu, L.; Chen, S.; Chen, G.; Ma, J Heat penetrability index (hpi) method for distilling subsurface thermal information from satellite infrared images. *Seismology and Geology* **2004**, *26*, 519–527.